

Sliding k -Transmitters: Hardness and Approximation

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Abstract

A *sliding k -transmitter* in an orthogonal polygon P is a mobile guard that travels back and forth along an orthogonal line segment s inside P . It can see a point $p \in P$ if the perpendicular from p onto s intersects the boundary of P at most k times. We show that guarding an orthogonal polygon P with the minimum number of k -transmitters is NP-hard, for any fixed $k > 0$, even if P is simple and monotone. Moreover, we give an $O(1)$ -approximation algorithm for this problem.

1 Introduction

Art gallery problems are one of the standard problems in computational geometry. In the original setting, we are given a polygon (modelling the art gallery) and we want to know a set of points (modelling guards or cameras) that can see any point in the polygon, where “see” in the original setting means that the line segment from the guard to the point is inside the polygon. There have been numerous results, concerning bounds on the number of guards needed, NP-hardness and approximation algorithms. See e.g. [13, 10] and the references therein.

Recently, motivated by covering a region with wireless transmitters, Aichholzer et al. [1] introduced variants where guards can see through a limited number of walls. Hence a *k -transmitter* is a point p in a polygon P that is considered to see all points q in P for which the line segment \overline{pq} intersects the boundary of P at most k times. Only cases of even k are interesting.

We combine in this paper the concept of a k -transmitter with the concept of a mobile guard. A *mobile guard* is a guard that is not stationary, but walks along a line segment s inside the polygon, and can see all points that are visible from some point of s . For orthogonal polygons, a common restriction has been to demand that line segment s is horizontal or vertical, and that it guards only those points p that it can see in an orthogonal fashion, i.e., the perpendicular from p onto s is inside P . This is called a *sliding camera*. We combine the concept of sliding cameras with k -transmitters, and hence define a *sliding k -transmitter* as follows: It is a horizontal or vertical line segment s inside an orthogonal polygon P and it can see all points p such that the

perpendicular from s onto p intersects the boundary of P at most k times. We allow sliding k -transmitters to include edges of the polygon.¹ The objective is to guard P with the minimum number of sliding k -transmitters.

Related Work. Sliding cameras were introduced by Katz and Morgenstern [9]. Finding the minimum set of sliding cameras is NP-hard in polygons with holes [8], even if only horizontal sliding cameras are allowed [4]. The optimum set of sliding cameras can be found in polynomial time for monotone polygons [7]. The complexity for simple polygons is open.

Finding the minimum set of k -transmitters is NP-hard in simple polygons [6], regardless whether the transmitters are points or polygon-edges. Numerous bounds are known on the number of k -transmitters that are necessary and sufficient, depending on the type of transmitter (point or edge) and the type of polygon [1, 2, 3, 6]. Regarding sliding k -transmitters, an approximation algorithm for monotone polygons is claimed in [12], but the algorithm needs a minor modification to deal with an example (private communication); it is not clear whether this modification suffices. Other optimization criteria for sliding k -transmitters have also been considered [11].

Our Results. In this paper, we study the complexity of finding the minimum set of sliding k -transmitters to guard an orthogonal polygon. Unsurprisingly, we can show that this is NP-hard, but we prove NP-hardness even in a very restricted version: The polygon is orthogonal and y -monotone, and there is an optimal solution with only horizontal sliding k -transmitters. We are not aware of *any* other variant of the art gallery problem that is NP-hard on orthogonal monotone polygons (the traditional art gallery problem is NP-hard for monotone polygons [10], but slanted edges are crucial for the reduction to work).

As a second result, we show that the $O(1)$ -approximation algorithm that we recently developed for sliding cameras [4] works similarly for sliding k -transmitters. Hence we have an $O(1)$ -approximation for finding the minimum set of sliding k -transmitters, in any (not necessarily simple) orthogonal polygon. The algorithm works also (and becomes even easier) if only horizontal sliding k -transmitters are allowed.

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¹With some minor modifications, the results in this paper also hold if guards must be strictly inside P except at their end.

2 NP-Hardness

In this section, we show that guarding with sliding k -transmitters is NP-hard, even if the polygon is orthogonal and monotone (hence simple). We first prove this for $k = 2$ and then extend to larger k .

2.1 Sliding 2-Transmitters

We use a reduction from Minimum Vertex Cover in a graph G , which is known to be NP-hard even if G is required to be planar and 2-connected (see e.g. [5]). So the objective is to compute a minimum set C of vertices such that every edge has at least one endpoint in C .

Given a planar 2-connected graph G with n vertices and m edges, we first compute a *bar visibility representation* of G in which each vertex is assigned a horizontal line segment (called *bar*) and for each edge there is a vertical *strip* with positive width that connects the bars of endpoints and does not intersect other vertices. It has been shown multiple times (see e.g. [14]) that this exists and can be computed in linear time. We may move vertex-bars up and down slightly as needed so that all vertex-bars have distinct y -coordinates. Also, since edge-strips have positive width, we can make them thin enough such that no two of them have overlapping x -range. Since the graph is 2-connected, the construction in [14] guarantees that all vertices except the bottom-most one have a neighbour below, and all vertices except the topmost one have a neighbour above.

Gadgets. We start by thickening each vertex-bar into a box, and place three copies of this box above each other with the same x -range. These three boxes are connected to each other by *channels*, which are thin vertical corridors (thin enough so that their x -range is strictly within that of the vertex-box, and does not intersect an edge-strip). We place these two channels at opposite ends of the vertex-boxes, resulting in a Z -shape or an S -shape (the choice between the two is arbitrary for now, but will be determined later). We call the result a *vertex-gadget*; see Fig. 1. By making the height of boxes small enough, we may assume that no two vertex-gadgets have overlapping y -range.

For each edge e , the *edge-gadget* of e is a small axis-aligned box placed strictly within the strip representing e in such a way that its y -range intersects no y -range of another (vertex- or edge-) gadget. See Fig. 1. Notice that from any edge-gadget there are vertical lines-of-sight to the vertex-gadgets of the endpoints of the edge.

The Reduction. Let P' be the polygon obtained by replacing all vertex-bars and edge-strips with these gadgets. P' is y -monotone (i.e., any horizontal line intersects it in one interval), but not connected (for now we allow the polygon to be disconnected, but we will discuss the modifications to make it connected later).

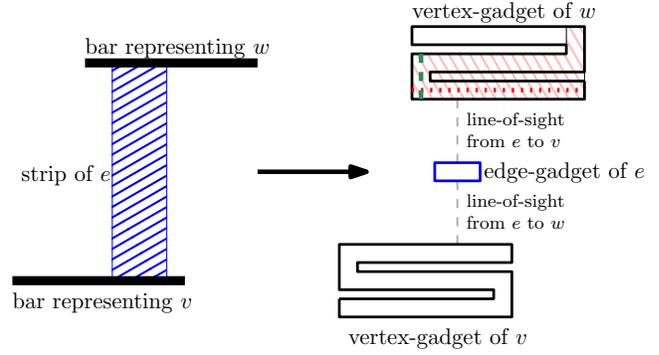


Figure 1: Vertex- and edge-gadgets. The pink (falling pattern) region is guarded by the red (dotted) horizontal 2-transmitter. Note that it includes everything that the green (dashed) vertical 2-transmitter can see.

Since no y -ranges overlap, one can easily verify that vertical 2-transmitters are never required.

Observation 1 *Any vertical sliding 2-transmitter in P' can be replaced by a horizontal sliding 2-transmitter that guards at least as much.*

See also Fig. 1. We call the three boxes of a vertex-gadget the *top*, *middle* and *bottom* box, and also use *outer boxes* to mean the top and bottom box.

Lemma 1 *For any set S of horizontal sliding 2-transmitters that guard P' entirely, there exists a set S' of horizontal sliding 2-transmitters that guard P' entirely such that $|S'| \leq |S|$ and no sliding 2-transmitter of S' is located in an edge-gadget.*

Proof. Let $s \in S$ be a sliding 2-transmitter that lies in an edge-gadget B corresponding to edge $e = (v, w)$. After possible renaming, assume that (the vertex-gadget corresponding to) v is below e and w is above e .

Assume first that one of v, w (say v) has a horizontal sliding 2-transmitter s' in the outer box facing e . After possibly extending s' we may assume that it spans the entire outer box of v . Since the x -range of B is within the x -range of v , s' sees everything that s saw and that was below s . So we can replace s by a sliding 2-transmitter in the outer box of w facing e , and this can only increase the guarded region.

So now assume that neither v nor w has a horizontal sliding 2-transmitter in the outer box facing e . Consider a point p in the top box of v that is just outside the x -range of B , but still within the x -range of w . The only horizontal sliding 2-transmitters that could guard p are in the bottom box of w or in the middle box of v . By assumption we therefore have a sliding 2-transmitter in the middle box of v . Likewise w must have a sliding 2-transmitter in the middle box of w . We can thus move the sliding 2-transmitter in B to the bottom box of w without decreasing the guarded region. \square

Lemma 2 *Let S be a set of horizontal sliding 2-transmitters that guard P' entirely and that do not lie in edge-gadgets. Then for any vertex v , there must be at least one sliding 2-transmitter intersecting the vertex-gadget of v . If there is exactly one such sliding 2-transmitter, then it must be in the middle box of v .*

Proof. Pick a point p in the middle box of v that is not in the x -range of the channels. Let s be a horizontal sliding 2-transmitter that guards p . Then s must be in one of the three boxes of v .

Assume now that exactly one sliding 2-transmitter intersects the vertex-gadget of v , and it is not in the middle box. Say the sliding 2-transmitter is in the bottom box. If v has any neighbour w above, then let p be a point in the top box of v and in the same x -range as the edge-gadget of (v, w) . To guard p , we need either a sliding 2-transmitter in the edge-gadget (which was excluded) or in the top or middle box of v (which was also excluded). So v cannot have any neighbour above. By construction that means that v is the topmost of all vertices. To guard the top box of v , we then must have a sliding 2-transmitter in the top or middle box of v . Again contradiction. \square

Lemma 3 *The following statements are equivalent: (i) G has a vertex cover of size k , (ii) P' can be guarded by $n + k$ sliding 2-transmitters, and (iii) P' can be guarded by $n + k$ horizontal sliding 2-transmitters.*

Proof. Given a vertex cover C of G , we place horizontal transmitters as follows: If $v \in C$, then place a maximal horizontal sliding 2-transmitter in both outer boxes of v , else place a maximal horizontal sliding 2-transmitter in the middle box of v . Clearly we have $n + |C|$ sliding 2-transmitters and every vertex-gadget is guarded. For every edge e , one endpoint v is in C , and hence both bottom and top box of v contain sliding 2-transmitters. The one in the outer box of v that faces e then guards the edge-gadget of e .

Vice versa, assume that set S of sliding 2-transmitters guards P' . By the above results, we may assume that they are all horizontal and none are in an edge-gadget. Define C to be all those vertices whose vertex-gadgets are intersected by at least two sliding 2-transmitters. Since every vertex-gadget intersects at least one sliding 2-transmitter we have $|C| \leq |S| - n$. For every edge (v, w) , the edge-gadget must be guarded by a sliding 2-transmitter that is in an outer box of v or w , say v . Then v must contain at least two sliding 2-transmitters by Lemma 2, so $v \in C$. Hence C is a vertex cover. \square

Connecting the Polygon. Now we explain how to make the polygon connected while staying monotone. Let g_1, \dots, g_{m+n} be the gadgets in P' , sorted in bottom-to-top order (since y -ranges are disjoint, this is well-defined). The idea is to connect each g_i to g_{i+1} using

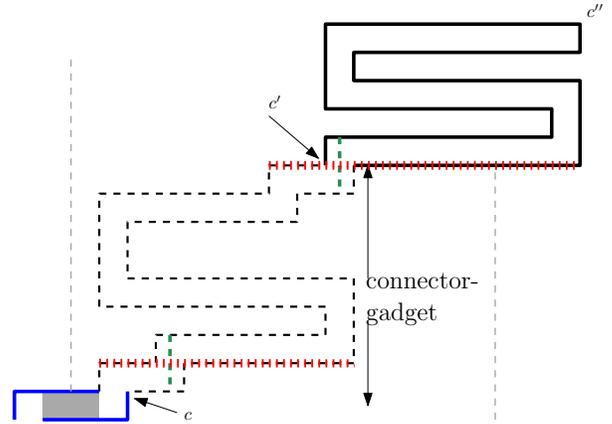


Figure 2: Connecting an edge-gadget to a vertex-gadget if there is no line of sight between them. We again show how some vertical transmitters can be replaced by horizontal transmitters.

a *connector-gadget*. This is an S -shaped or Z -shaped gadget much like a vertex-gadget, except that the top and bottom box both add a zig-zag near the end. Also, one of the channels has flexible height, so that the connector-gadget can have arbitrary height. We attach the ends of the connector-gadget C to corners of g_i and g_{i+1} . Fig. 2 shows how to do this if the x -range of C is disjoint (except at the ends) from the ones of g_i and g_{i+1} , and the inset in Fig. 3 shows how to do this if C shares x -range with them (in case of which we push the zig-zag to the very end to avoid overlap.)

However, we cannot connect consecutive gadgets if the connector-gadget would cross a line-of-sight. To avoid doing this, we will subdivide edges.

Observation 2 (Folklore) *If G^s results from graph G by subdividing one edge twice, then G has a vertex cover of size k if and only if G^s has a vertex cover of size $k+1$.*

We proceed as follows. First “parse” the bottommost gadget g_1 : use an S -shape for it and fix as current corner its top right corner. Assume now we have parsed gadget g_i already, and fixed one top corner c of it as current corner. Let g_{i+1} be the next gadget above g_i . Considering its two bottom corners, we choose the corner c' so that $\overline{cc'}$ crosses as few lines-of-sight as possible.

If line segment $\overline{cc'}$ crosses no line-of-sight, then attach a connector-gadget between c and c' , using as shape (i.e., S or Z) the one that has c and c' at its ends. Let c'' be the diagonally opposite corner from c' in gadget g_{i+1} and (if g_{i+1} is a vertex-gadget) use as shape (i.e., S or Z) for it the one that has c' and c'' at its ends. This finishes (in this case) the parsing of gadget g_{i+1} , and we continue to connect to the next gadget with current corner c'' .

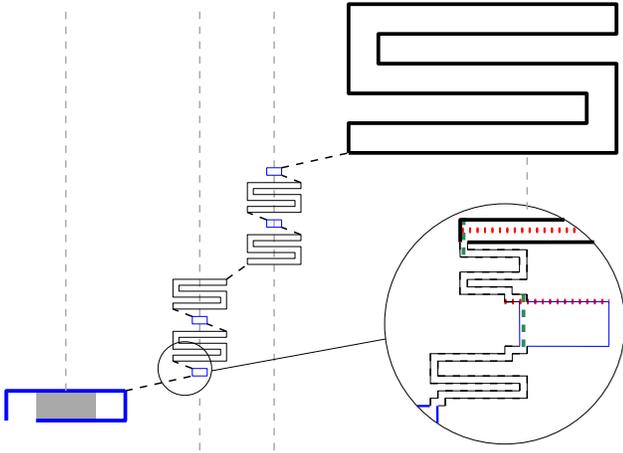


Figure 3: Connecting an edge-gadget to a vertex-gadget if there are lines of sight between them.

Now assume that $\overline{cc'}$ crosses some lines-of-sight, say l_1, \dots, l_ℓ in order from c to c' . For all j , line-of-sight l_j represents an edge e_j ; subdivide e_j twice. This adds two new vertex-gadgets and two new edge-gadgets that we place along l_j , in the y -range between g_i and g_{i+1} . We make their height small enough and move them up and down suitably (while staying between g_i and g_{i+1}), so that all their y -ranges are disjoint and the ones of l_j are below the ones of l_{j+1} for all j .

All these gadgets can be connected with line segments that do not cross a line-of-sight. See Fig. 3. We can hence connect all these gadgets as explained above. The only difference is that the next current corner c'' must be chosen to be the end of the line segment connecting to the next gadget. Normally c'' will again be diagonally opposite from the previous corner c' , but there is one exception per set of gadgets added for subdivisions. With that we have connected to g_{i+1} , and we repeat from there (after choosing its shape and the current corner as before).

Reduction Revisited. With the addition of connector-gadgets, Observation 1 (vertical transmitters can be replaced by horizontals) is not as obvious anymore, but still holds as long as sliding k -transmitters may run along polygon-edges. See Fig. 2. With this, Lemma 1, Lemma 2, and the equivalent of Lemma 2 for connector-gadgets, also hold. Let N_s be the total number of subdivisions that we did over all connecting of all gadgets (N_s is even), and let G' be the graph that results. We started with $n+m \in O(n)$ vertex-gadgets and edge-gadgets and $2m \in O(n)$ lines-of-sight. Connecting two of these gadgets hence creates $O(n)$ subdivisions, and therefore $N_s \in O(n^2)$ is polynomial. After all subdivisions we have $n+m+2N_s$ gadgets, and hence use $N_c := n+m+2N_s-1$ connector-gadgets to connect all of them into one polygon P . So the construction is

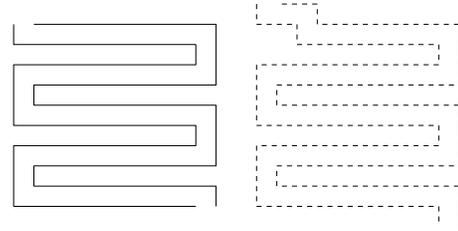


Figure 4: Vertex- and connector-gadget for $k = 4$.

polynomial. G has a vertex cover of size k if and only if G' has a vertex cover of size $k' := k + N_s/2$ if and only if P can be guarded with $k' + n + N_c$ horizontal sliding 2-transmitters.

With that, the reduction is complete for $k = 2$. Note that the constructed polygon is connected and y -monotone (and in particular therefore simple).

2.2 k -Transmitters for $k > 2$

We now generalize to sliding k -transmitters for any fixed $k > 0$. The reduction is exactly the same as before, with the exception of the definition of vertex-gadgets and connector-gadgets.

The vertex-gadget now consists of $k + 1$ copies of the thickened bar in the visibility representation (earlier we had $3 = 2 + 1$ copies). They are connected with k channels at alternate ends, resulting in a zig-zag line. The connector-gadget is a vertex-gadget with additional small zig-zags in the top and bottom box (possibly pushed towards the end.) See Fig. 4.

We can verify that again vertical sliding k -transmitters are never better than horizontal ones. Define for a vertex-gadget the *middle* box to be the $(k/2+1)$ st box (recall that k is even), and the *outer boxes* to be the top and bottom box as before. With that, the proofs of Lemmas 1 and 2 carry almost verbatim, and the reduction holds again. We conclude:

Theorem 4 *For any $k > 0$, guarding a polygon with the minimum set of sliding k -transmitters is NP-complete, even if (i) the polygon is a simple y -monotone orthogonal polygon, and (ii) only horizontal sliding k -transmitters are allowed.*

Notice that every gadget is a *thickened path* obtained by sliding a unit square along an orthogonal path. With suitable rescaling, in fact the entire polygon can be made into a thickened path, with one exception: Whenever we subdivide edges, we must (at one edge-gadget) attach both connecting gadgets on the same (left or right) side, hence have a “leg” sticking out. (This could perhaps be called a *thickened caterpillar*.) We suspect that the construction could be modified to become a thickened path, but have not been able to work out the details yet.

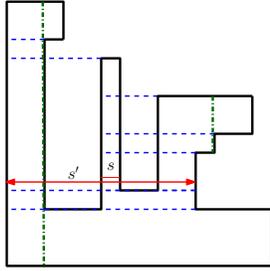


Figure 5: Horizontal partition-segments (blue dashed), one horizontal slice-segment (red solid) and two vertical guard-segments (green dot-dashed) for $k = 2$.

3 An $O(1)$ -Approximation Algorithm

In this section, we give an $O(1)$ -approximation algorithm for the sliding k -transmitter problem, using as key ingredient an $O(1)$ -approximation developed in [4] for a certain hitting problem among segments. The main difference between our approach and the one in [4] is that we need to define the segments differently so that we encapture that guards can see through k walls.

Let P denote an orthogonal polygon with n vertices, and let δP denote the boundary of P . We first compute a subdivision of P and then define sets of orthogonal line segments in P which will be used to define the hitting set problem.

Slices. Define *horizontal partition-segments* as follows: Start with a horizontal edge e . Expand e leftwards until we hit a vertical edge of the polygon, coming from the strict inside of P , for the $(k/2)$ th time. Likewise expand e rightwards. If there are not enough such intersections, then stop at the last one. See Fig. 5.

The horizontal partition-segments split the interior of the polygon into rectangles that we call *horizontal slices*. Since any edge gives rise to one partition-segment, and any partition-segment intersects $O(k)$ vertical edges, we have $O(kn)$ horizontal slices.

The following lemma argues that this partitioning is “correct” in the sense that any transmitter either guards all or nothing of the interior of a slice.

Lemma 5 *Let σ be a horizontal slice and let c be a point in its interior. If a maximal vertical sliding k -transmitter g sees c , then it sees all points of σ .*

Proof. Let q be the point on g where the perpendicular from c onto g ends. By assumption the horizontal line segment \overline{cq} intersects the boundary of P at most k times. Expand \overline{cq} until it spans the x -range of σ ; this cannot add crossings since $\sigma \subseteq P$. Now sweep the resulting segment s' upward until we hit either the top side of σ or a horizontal edge of P . Say we hit an edge e first. We created a partition-segment from e , which extends in both directions until it hits at most $(k/2)$ vertical

edges from the inside, hence at most k vertical edges. This partition segment contains the entire (translated) s' and splits B , so we have reached the top side of B .

So we can sweep the region of B above s' without encountering new edges of P , which shows that g guards all of this. Likewise we can sweep downward until the bottom side of B , and so g guards all of B . \square

Slice-segments. We now assign a segment to each horizontal slice that captures “being guarded”. For any horizontal slice σ , let s be a horizontal segment strictly inside σ . Extend s (much like we did for partition-segments) to both sides until it hits a vertical edge from the inside for the $(k/2)$ th time. We call the resulting segment s' the *slice-segment* of σ . See also Fig. 5.

We define vertical slices of P and vertical slice-segments in an analogous fashion. There are $O(kn)$ slice-segments since there are $O(kn)$ slices.

Guard-segments Γ . Our definition of sliding k -transmitters allowed any horizontal or vertical segment to be used as such. We now describe a finite set of sliding k -transmitters and argue that these suffice. Let s be a horizontal edge of P . Define a sliding k -transmitter s' obtained by extending s until we hit an interior point of a vertical edge of δP . (If some vertices are aligned, then s' may run along multiple horizontal edges of P .) The resulting segments are the *horizontal guard-segments* Γ_H . Define *vertical guard-segments* Γ_V similarly, and set $\Gamma = \Gamma_H \cup \Gamma_V$ to be the guard-segments. We have at most n guard segments (one per edge).

Crosses X : Let a *pixel* be any rectangle that has the form $\sigma_H \cap \sigma_V$ for a horizontal slice σ_H and vertical slice σ_V . Let c be the point where the slice-segments s_H, s_V corresponding to σ_H and σ_V intersect; we call c a *cross*, and say that s_H and s_V *support* c . Note that c is in the interior of the pixel since slice-segments were defined using segments strictly in the interior of the slice. We denote the set of crosses by X .

We say that a cross c is *hit* by a guard-segment g if g intersects the supporting slice-segment of c that is perpendicular to g . We now show that reducing the problem to just crosses and guard-segments is enough.

Lemma 6 *A set S of m sliding k -transmitters guards P if and only if there exists a set $S' \subseteq \Gamma$ of m guard-segments such that every cross c is hit by some guard-segment $\gamma \in S'$.*

Proof. (\Rightarrow) Suppose that we have a set S of m sliding k -transmitters that guards P entirely. Fix one sliding k -transmitter s . Translate s in parallel (i.e., move it horizontally if s is vertical, move it vertically if s is horizontal) until we reach δP . Thus s is now intersecting an edge of P . Extend s so that it is maximal while still within P . Both operations can only increase the region

seen. The resulting segment s' is a guard-segment. After doing this to all sliding k -transmitters, we now have a set of guard-segments S' that sees all of P . Now consider any cross $c \in X$. Since c is a point in P , it is guarded by some guard-segment $\gamma \in S'$. Thus, there exists a point $g \in \gamma$ such that the line segment gc is normal to γ and intersects δP in at most k points. But, gc is part of the slice-segment that supports c and is perpendicular to γ . So g is the intersection point between that slice-segment and guard-segment γ .

(\Leftarrow) This is straightforward by Lemma 5 since (i) any point in P belong to at least one pixel, (ii) there is a 1-to-1 correspondence between the pixels and crosses of P , and (iii) crosses are interior points of pixels. \square

Our problem has now been discretized as follows: Given the set X of all crosses, each supported by two line segments, find a subset $S \subseteq \Gamma$ such that for every cross one of the two line segments is intersected by at least one guard-segment in S . We call this the *cross-hitting* problem. This problem is *exactly* the same problem as studied in [4] when solving the sliding-cameras problem (the only difference is in the choice of supporting segments of crosses, which are longer for sliding k -transmitters). They give an $O(1)$ -approximation algorithm for this problem which uses no information about how the segments were obtained (other than that they are horizontal or vertical). Using this, we hence have:

Theorem 7 *For any $k > 0$, there exists a polynomial-time $O(1)$ -approximation algorithm for guarding an orthogonal polygon with sliding k -transmitters.*

As in [9], we also consider the variant when only horizontal sliding k -transmitters are allowed. This also reduces to the cross-hitting problem, with the only change that we use Γ_H in place of Γ . This in fact simplifies the problem, because now only vertical supporting segments are relevant for crosses. So there is also an $O(1)$ -approximation algorithm for guarding an orthogonal polygon with horizontal sliding k -transmitters.

4 Conclusion

In this paper, we studied how to guard an orthogonal polygon using the minimum number of sliding k -transmitters. We showed that this is NP-hard, even if the polygon is y -monotone. We also gave an $O(1)$ -approximation algorithm.

The main open problem is to find better approximation factors. (The “ $O(1)$ ” in [4] stems from the use of ε -nets, and the constant is unspecified but likely quite large.) Is the problem APX-hard? Also, for what subclass of polygons is guarding with sliding k -transmitters polynomial? This is true for orthogonally convex polygons (one or two guards are always enough), but are there other, less trivial classes?

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